Network Analysis (Subject Code: 06ES34)

Transient Behavior and Initial Conditions

- Introduction
- Initial Conditions in Networks
- V-I Relationship of Network Elements
- Initial Conditions in Network Elements
- Procedure for Evaluating Initial Conditions
- Solution of Initial Value Problems

I. Initial Conditions in Networks:

There are many reasons for studying initial (and final) conditions;
(i) The most important reason is that initial conditions must be known to evaluate the arbitrary constants that appear in the general solution of the differential equations
(ii) The initial conditions give knowledge of the behavior of the elements at the instant of switching

At reference time $t=0$, the switch is closed (we assume that switch act in zero time). To differentiate between the time immediately before and immediately after the operation of a switch, we will use $-$ve and $+$ve signs. Thus conditions existing just before the switch is operated will be designated as $i\ (0^-)$, $v\ (0^-)$ etc. Conditions after as $i\ (0^+)$, $v\ (0^+)$ etc. Initial conditions in a network depend on the past history of the network prior to $t = 0^-$ and the network structure at $t = 0^+$, after switching. The evaluation of all voltages and currents and their derivatives at $t = 0^+$, constitutes the evaluation of initial conditions. Some times we use conditions at $t = \infty$; these are known as final conditions

II. V-I Relationships of Network Elements:

(i) Resistor:

$$v(t)= R \ i(t) \quad \text{and} \quad i(t) = \frac{v(t)}{R}$$

(ii) Inductor:

$$v(t)= L \ \frac{di(t)}{dt} \quad \text{and} \quad i(t) = \int_0^t v(t) \ dt$$

$$v(t)= L \ \frac{di(t)}{dt} \quad \text{and} \quad i(t) = \int_0^t v(t) \ dt + i_L(0^-)$$
(iii) Capacitor:

\[ v(t) = \frac{1}{C} \int_{0^-}^{t} i(t) \, dt \quad \text{and} \quad i(t) = C \frac{dv(t)}{dt} \]

\[ v(t) = \frac{1}{C} \int_{0^-}^{t} i(t) \, dt + V_C (0^-) \quad \text{and} \quad i(t) = C \frac{dv(t)}{dt} \]

III. Initial Conditions in Network Elements:

(i) Resistor: In resistor, current and voltage are related by ohm’s law \( v(t) = R \cdot i(t) \). From this relation, the current through a resistor will change instantaneously if the voltage changes instantaneously.

(ii) Inductor:

When switch is closed at \( t = 0 \), the current through an inductor cannot change instantaneously. As a result, closing of a switch to connect an inductor to a source of energy will not cause current to flow at that instant and inductor will act as an open circuit.

If a current of \( I_0 \) amps flows in the inductor at the instant of switching takes place, that current will continue to flow & for the initial instant the \( (t=0^+) \) inductor can be considered as a current source of \( I_0 \) amps

\[ i(t) = \frac{1}{L} \int_{-\infty}^{t} v(t) \, dt = \frac{1}{L} \int_{-\infty}^{0^-} v(t) \, dt + \frac{1}{L} \int_{0^-}^{t} v(t) \, dt \]

\[ i(t) = i(0^-) + \frac{1}{L} \int_{0^-}^{t} v(t) \, dt \quad \text{putting} \quad t = 0^+ \quad \text{on both sides} \]
\[
0^+ \\
i(0^+) = i(0^-) + \left(\frac{1}{L}\right) \int v(t) \, dt \quad 0^-
\]

Therefore \(i(0^+) = i(0^-)\)

If \(i(0^-) = 0\), then \(i(0^+) = 0\) This means that inductor acts as an open circuit

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The final condition (steady state condition) equivalent circuit of an inductor is derived from the basic relationship \(v = L \frac{di}{dt}\)

Under steady state condition \(\frac{di}{dt} = 0\)

This means \(v = 0\) and hence \(L\) acts as a short circuit at \(t = \infty\) (final or steady state). The final condition equivalent circuits of an inductor is shown in figure

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(iii) Capacitor:

At \(t = 0\), switch is closed

We have,

\[
\begin{align*}
  v(t) &= \frac{1}{C} \int i(t) \, dt \\
  v(t) &= \left(\frac{1}{C}\right) \int i(t) \, dt + \frac{1}{C} \int_0^t i(t) \, dt \\
  v(t) &= v(0^-) + \frac{1}{C} \int i(t) \, dt
\end{align*}
\]
Evaluating the expression at \( t = 0^+ \), we get
\[
v(0^+) = v(0^-) + \frac{1}{C} \int_{0^-}^{0^+} i(t) \, dt
\]
Therefore \( v(0^+) = v(0^-) \)
Thus the voltage across a capacitor cannot change instantaneously.
If \( v(0^-) = 0 \), then \( v(0^+) = 0 \). This means that \( t = 0^+ \), capacitor acts as a short circuit.
If \( v(0^-) = q_0/C \), then \( v(0^+) = q_0/C \). This means that \( t = 0^+ \), capacitor acts as a voltage source.

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**Element and Initial Condition**

**Equivalent Circuit at \( t=0^+ \)**

The final condition (steady state condition) equivalent circuit of an inductor is derived from the basic relationship \( i(t) = \frac{dv(t)}{dt} \)
Under steady state condition \( dv(t)/dt = 0 \). i.e. at \( t = \infty \), \( i(t) = 0 \) this means that, \( t = \infty \), (final or steady state) capacitor acts as an open circuit. The final condition equivalent circuits of a capacitor is shown in figure

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**Element and Initial Condition**

**Equivalent Circuit at \( t=\infty \)**

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**IV. Procedure for Evaluating Initial Conditions:**

There is no unique procedure that must be followed in solving for initial conditions.
We usually to solve for initial values of currents and voltages, an equivalent network of the original network at \( t = 0^+ \) is constructed according to the following rules;
(i) Replace all inductors with open circuits or with current source having source current equal to that flowing at time \( t=0^+ \)
(ii) Replace all capacitors with short circuits or with a voltage source of value \( v_0 = q_0/C \) if an initial charge \( q_0 \).
(iii) Resistors are left in the network without change.
**Step 1:** Solve the initial values of variables namely currents, voltages and charge at $t=0^+$

**Step 2:** Solve the initial derivatives of variables at $t=0^+$

**V. Problems:**

[1] In figure below, the switch ‘S’ is closed at $t=0$. Find the initial conditions $i(0^+)$ and $di(0^+)/dt$

![Circuit Diagram](image1)

**Solution:** At $t=0^+$, the equivalent circuit is

![Equivalent Circuit](image2)

Therefore $i(0^+)=0$

Applying KVL to the given circuit, $V = L \frac{di(t)}{dt} + RI(t)$

At $t=0^+$,

$L \frac{di(0^+)}{dt} + R i(0^+) = V$ since $i(0^+)=0$

$\frac{di(0^+)}{dt} = \frac{V}{L}$

[2] In the network of figure below, If $t=0$, switch ‘k’ is closed. Find the values of $i$, $di/dt$ and $d^2i/dt^2$ at $t=0^+$ for element values as follows; $V=100V$, $R=1000\Omega$ and $L=1H$.

![Circuit Diagram](image3)

**Solution:** When $t=0$, switch ‘k’ is closed. Then, after closing of switch, circuit becomes

![Equivalent Circuit](image4)

Therefore $i(0^+)=0$
Applying KVL to the given circuit, we have,

\[ 100 = R i(t) + L \frac{di(t)}{dt} \]  

At \( t = 0^+ \)

\[ 100 = R i(0^+) + L \frac{di(0^+)}{dt} \]  

Substituting the values of \( L, R, \) and \( i(0^+) \), we get

\[ 100 = 1000 \times 0 + 1 \times \frac{di(0^+)}{dt} \]

Therefore \( \frac{di(0^+)}{dt} = 100 \text{ Amps/sec} \)

Similarly, to find out second derivatives of the current, differentiate equation (2) with respect to \( t \).

\[ 0 = R \frac{di(0^+)}{dt} + L \frac{d^2i(0^+)}{dt^2} \]

\[ 0 = 1000 \times 100 + 1 \frac{d^2i(0^+)}{dt^2} \]

Therefore \( \frac{d^2i(0^+)}{dt^2} = -10^5 \text{ Amps/sec}^2 \)

[3] Consider the R-C circuit shown below, switch ‘S’ is closed at \( t=0 \) and assume that there is no initial charge in the capacitor. Find the initial conditions \( i(0^+) \) and \( \frac{di(0^+)}{dt} \)

![R-C Circuit Diagram]

Solution: At \( t = 0^+ \), the equivalent circuit is

When \( t = 0^+ \), capacitor acts as a short circuit. Therefore \( i(0^+) = \frac{v(t)}{R} \)

Applying KVL to the given circuit, we get

\[ v(t) = i(t)R + C \frac{di(t)}{dt} \]  

Differentiating equation (1), with respect to \( t \), we get

\[ 0 = R \frac{di(t)}{dt} + \frac{i(t)}{C} \]

At \( t = 0^+ \),

\[ 0 = R \frac{di(0^+)}{dt} + \frac{i(0^+)}{C} \]

\[ R \frac{di(0^+)}{dt} = - \frac{i(0^+)}{C} \]

\[ \frac{di(0^+)}{dt} = - \frac{i(0^+)}{RC} \]

\( \frac{di(0^+)}{dt} = - \frac{v}{R^2C} \)  

(since \( i = \frac{v}{R} \)
[4] In the given circuit, switch ‘K’ at t=0 is closed with the capacitor uncharged. Find the values of (i) $\frac{di}{dt}$ and (ii) $\frac{d^2i}{dt^2}$ at $t = 0^+$

Solution: When $t=0$, the switch ‘K’ is closed. Then circuit becomes

At $t = 0^+$, $i(0^+) = \frac{V}{R} = \frac{10}{1000} = 0.01$ Amp.

Applying KVL to the given circuit, we get

$$V(t) = R\, i(t) + \frac{1}{C} \int i(t) dt$$  \hspace{1cm} (1)

Differentiate equation (1) with respect to $t$, we get

$$0 = R \, \frac{di(t)}{dt} + \frac{i(t)}{C}$$

At $t = 0^+$, $R \, \frac{di(0^+)}{dt} = -\frac{i(0^+)}{C}$

$$\frac{di(0^+)}{dt} = -\frac{i(0^+)}{RC} = -\frac{0.01}{1000 \times 1 \times 10^{-6}} = -10 \text{ Amps/sec}$$

Differentiate equation (1) twice with respect to $t$, we get

$$R \, \frac{d^2i(0^+)}{dt^2} + \frac{1}{C} \, \frac{di(0^+)}{dt} = 0$$

At $t = 0^+$, $\frac{d^2i(0^+)}{dt^2} = -(\frac{1}{CR}) \frac{di(0^+)}{dt} = -(\frac{1}{1000 \times 1 \times 10^{-6}}) \times (-10) = 10000 \text{ Amps/sec}^2$

[5] For the given circuit, find $i(0^+)$, $\frac{di(0^+)}{dt}$ and $\frac{d^2i(0^+)}{dt^2}$ when $t=0$ switch ‘K’ is closed. Initially, Inductor having zero current and capacitor having zero charge.
Solution: When t=0, switch ‘K’ is closed. Then circuit becomes

\[ V(t) = R \dot{i}(t) + L \frac{d^2i(t)}{dt^2} + \frac{1}{C} \int \! \dot{i}(t) \, dt \]  

Therefore \( i(0^+) = 0 \)

Applying KVL

\[ V(t) = R \dot{i}(t) + L \frac{d^2i(t)}{dt^2} + \frac{1}{C} \int \! \dot{i}(t) \, dt \]  

Since \( V_c(0^-) = 0 \) Therefore \( \frac{1}{C} \int \! \dot{i}(t) \, dt = 0 \)  

(because \( q_0 = 0 \))

At \( t = 0^+ \), the inductor will act as an open circuit and capacitor will act as a short circuit.

\[ V(t) = 0 + L \frac{d^2i(0^+)}{dt^2} + 0 \]

Therefore \( \frac{di(0^+)}{dt} = \frac{V(t)}{L} \text{ Amps/sec} \)

Differentiate equation (1) with respect to t, we get

\[ 0 = R \frac{d^2i(t)}{dt^2} + L \frac{d^2i(t)}{dt^2} + \frac{i(t)}{C} \]  

Therefore \( \frac{d^2i(0^+)}{dt^2} = \frac{-RV(t)}{L^2} \text{ Amps/sec}^2 \)

[6] In the given circuit, switch ‘K’ is closed at \( t=0 \) with capacitor uncharged and zero current in the inductor. Find \( \frac{di(t)}{dt} \) and \( \frac{d^2i(t)}{dt^2} \) at \( t = 0^+ \)

\[ \text{Solution: At } t=0, \text{ the switch ‘K’ is closed. Then circuit becomes} \]

\[ V(t) = R \dot{i}(t) + L \frac{d^2i(t)}{dt^2} + \frac{1}{C} \int \! \dot{i}(t) \, dt \]

Therefore \( i(0^+) = 0 \)

At \( t=0^+ \), the inductor will act as an open circuit and capacitor will act as a short circuit.
Applying KVL to the given circuit at time $t=0^+$

$$V(t) = R \frac{di(t)}{dt} + L \frac{d^2i(t)}{dt^2} + 1/C \int i(t) dt$$  \hspace{1cm} (1)

At $t =0^+$, equation (1) becomes

$$V(0^+) = R i(0^+) + L \frac{di(0^+)}{dt} + 1/C \int i(0^+) dt$$

$$100 = 100x0 + 1x \frac{di(0^+)}{dt} + (1/1x10^{-6}) \int 0 dt$$

Therefore $\frac{di(0^+)}{dt} = 100 \text{ Amps/sec}$

Differentiate equation (1) with respect to $t$, we get

$$0 = R \frac{di(t)}{dt} + L \frac{d^2i(t)}{dt^2} + i(t)/C$$  \hspace{1cm} (2)

At $t =0^+$, equation (2) becomes

$$0 = R \frac{di(t)}{dt} + L \frac{d^2i(0^+)}{dt^2} + 0$$

Therefore $\frac{d^2i(0^+)}{dt^2} = - (R/L) \frac{di(t)}{dt} = (100/1)x100 = -10000 \text{ Amps/sec}^2$

[7] In the network shown below, the switch ‘K’ is opened at $t=0$ after the network has attained a steady state with the switch closed. Find (a) the expression for the voltage across the switch at $t=0^+$ (b) If the parameters are adjusted such that $i(0^+)=1$ and $\frac{di(t)}{dt} = -1$, what is the value of the derivative of the voltage across the switch $\frac{dV_k(0^+)}{dt} =$?

Solution:

(a) Before opening the switch ‘K’, circuit is

Therefore $i(0^+) = V/R_2$
After opening the switch $k$, circuit is

\[ V_{k} = R_1 x i + \frac{1}{C} \int i dt \]  \hspace{1cm} (1)

At $t=0^+$, $i(0^-) = V/R_2$

The voltage across the switch $V_k = R_1 x i + (1/C) \int i dt$

At $t=0^+$, $(1/C) \int i dt = 0$

Therefore $V_k = R_1 x i(0^+) = R_1 x V/R_2 = Vx(R_1/R_2)$

(b) Differentiate equation (1) w. r. t. t, we get

\[ \frac{dV_k}{dt} = R_1 \frac{di}{dt} + i/C \]

\[ \frac{di(0^+)}{dt} = -1; \quad i(0^+) = 1 \] are given

At $t=0^+$

\[ \frac{dV_k(0^+)}{dt} = R_1 \frac{di(0^+)}{dt} + i(0^+)/C \]

\[ = R_1(-1) + 1/C = (1/C) - R_1 \]

[8] The following figure with switch ‘k’ is closed, steady state has been reached. At $t=0$, the switch is open. Find $V_k(0^+)$ and $\frac{d^2V_k(0^+)}{dt^2}$

Solution: Just before opening the switch ($t<0$) i.e. at $t=0^-$, The circuit becomes
Therefore $i(0^-) = 2/1 = 2 \text{ Amps} = i(0^+)$

At $t=0^+$ switch ‘k’ is opened, the circuit becomes

Apply KVL to the circuit, we get

\[ V = \frac{1}{C} \int i \, dt + L \frac{di}{dt} + R x i \]  

(1)

At $t=0^+$,

\[ 2 = 0 + 1 \frac{di(0^+)}{dt} + 1 \times i(0^+) \]

\[ \frac{di(0^+)}{dt} = 2 - 1 \times i(0^+) = 0 \text{ Amps/sec} \]

Differentiate equation (1) w. r. t. t, we get

\[ 0 = \frac{i}{C} + L \frac{d^2i}{dt^2} + R \frac{di}{dt} \]

At $t=0^+$,

\[ \frac{d^2i(0^+)}{dt^2} = -(\frac{R}{L}) \frac{di(0^+)}{dt} - i(0^+)/LC \]

\[ = \frac{1}{1} \frac{di(0^+)}{dt} - i(0^+)/[1x(1/2)] = 0 - 2 \times 2 = -4 \text{ Amps/sec}^2 \]

Differentiate equation (1) twice w. r. t. t, we get

\[ 0 = \frac{1}{C} \frac{di}{dt} + L \frac{d^3i}{dt^3} + R \frac{d^2i}{dt^2} \]

At $t=0^+$,

\[ \frac{d^3i(0^+)}{dt^3} = -2 \frac{di(0^+)}{dt} - \frac{d^2i(0^+)}{dt^2} \]

\[ = -2 \times 0 - (-4) = 4 \text{ Amps/sec}^3 \]

From (1) $V_k + L \frac{di}{dt} + R x i = 2$ (since $\frac{1}{C} \int i \, dt = V_k$)

Differentiate above equation twice w. r. t. t, we get

\[ \frac{d^2 V_k}{dt^2} + L \frac{d^3i}{dt^3} + R \frac{d^2i}{dt^2} = 0 \]

At $t=0^+$,

\[ \frac{d^2 V_k(0^+)}{dt^2} = -L \frac{d^3i(0^+)}{dt^3} - R \frac{d^2i(0^+)}{dt^2} \]

\[ = -1 \times 4 - 1 \times (-4) = 0 \text{ Volts/sec}^2 \]
[9] In the given circuit, switch ‘k’ is opened at t=0. Find the values of v, dv/dt, d²v/dt² at t = 0⁺.

Solution: At t = 0⁻

\[ v(0^-) = 0 \]
\[ i_L(0^-) = 0 \]
\[ i_L(0^+) = 0 \text{ since current through the inductor cannot change instantaneously} \]

At t = 0⁺

\[ v(0^+) = i \times R = 2 \times 1000 = 2000 \text{V} \]

Apply KCL to the given circuit at t = 0⁺, we get

\[ \frac{v}{R} + \frac{1}{L} \int v \, dt = 2 \]

(1)

Differentiate equation (1) with respect to t, we get

\[ \frac{1}{R} \frac{dv}{dt} + \frac{1}{L} v = 0 \]

(2)

At t = 0⁺, equation (2) becomes

\[ \frac{1}{R} \frac{dv(0^+)}{dt} + \frac{1}{L} v(0^+) = 0 \]

\[ \frac{dv(0^+)}{dt} = - \frac{(R/L)v(0^+)}{dt} \]

\[ = - \frac{1000}{1} \times 2000 = -2 \times 10^6 \text{ volts/sec} \]

Differentiate equation (2) with respect to t, we get

\[ \frac{1}{R} \frac{d^2v}{dt^2} + \frac{1}{L} \frac{dv}{dt} = 0 \]

(3)

At t = 0⁺, equation (3) becomes

\[ \frac{1}{R} \frac{d^2v(0^+)}{dt^2} + \frac{1}{L} \frac{dv(0^+)}{dt} = 0 \]

\[ \frac{d^2v(0^+)}{dt^2} = - \frac{(R/L)dv(0^+)/dt}{dt} = - \frac{(1000/1) \times (-2 \times 10^6)}{dt} = 2 \times 10^9 \text{ volts/sec}^2 \]
[10] In the given circuit, switch ‘k’ is opened at t=0. Find the values of v, dv/dt, d²v/dt² at t = 0⁺.

Solution: At t = 0⁻

At t = 0⁺, v (0⁺) = 0 since voltage across the capacitor cannot change instantaneously.

Apply KCL to the given circuit at t = 0⁺, we get

\[ \frac{V}{R} + C \frac{dV}{dt} = 10 \] \hspace{1cm} (1)

At t = 0⁺, equation (1) becomes

\[ \frac{dV(0⁺)}{dt} = \frac{10}{C} - \frac{v(0⁺)}{RC} \]

\[ = \frac{10}{(1\times10^{-6})} - 0 = 10^7 \text{ Volts/sec} \]

Differentiate equation (1) with respect to t, we get

\[ \frac{1}{R} \frac{dV}{dt} + CD \frac{d^2V}{dt^2} = 0 \]

At t = 0⁺, equation (2) becomes

\[ CD \frac{d^2V(0⁺)}{dt^2} = - \left( \frac{1}{R} \right) \frac{dV(0⁺)}{dt} \]

\[ d^2V(0⁺) = - \left( \frac{1}{RC} \right) \frac{dv(0⁺)}{dt} = - \left[ \frac{1}{(100\times1\times10^{-6})} \times 10^7 \right] \]

\[ = -10^{11} \text{ volts/sec}^2 \]
[11] In the given circuit, switch ‘k’ is opened at \( t=0 \). Find the values of \( v_1 \), \( v_2 \), \( \frac{dv_1}{dt} \) and \( \frac{dv_2}{dt} \) at \( t = 0^+ \).

\[ \text{Solution: At } t = 0^- \]

\[
\begin{align*}
   v_1(0^-) &= 0 \\
   v_2(0^-) &= 0
\end{align*}
\]

At \( t = 0^+ \),

\[
\begin{align*}
   v_1(0^+) &= iR_1 = 10 \times 10 = 100 \text{V} \\
   v_2(0^+) &= 0 \text{ and } i_2(0^+) = 0
\end{align*}
\]

Apply KCL to the given circuit at \( t = 0^- \), we get

\[
\begin{align*}
   v_1/R_1 + (1/L) \int (v_1 - v_2) \, dt &= 0 \\
   v_2/R_2 + (1/L) \int (v_2 - v_1) \, dt + c \frac{dv_2}{dt} &= 0
\end{align*}
\]

(1) (2)

Differentiate equation (1) with respect to \( t \), we get

\[
\begin{align*}
   (1/R_1) \frac{dv_1}{dt} + (1/L)(v_1 - v_2) &= 0 \\
   \text{At } t = 0^+, \\
   (1/R_1) \frac{dv_1(0^+)}{dt} + (1/L)v_1(0^+) - (1/L)v_2(0^+) &= 0
\end{align*}
\]

\[
\frac{dv_1(0^+)}{dt} = \frac{-R_1/L}{v_1(0^+) + (R/L)v_2(0^+)}
\]

\[
= -\frac{(10/1) \times 100}{(10/1) \times 0} = -1000 \text{ volts/sec.}
\]

From equation (2)

\[
\begin{align*}
   v_2(0^+)/R_2 + i_L(0^+) + c \frac{dv_2(0^+)}{dt} &= 0 \\
   0 + 0 + c \frac{dv_2(0^+)}{dt} &= 0 \\
   c \frac{dv_2(0^+)}{dt} &= 0 \text{ volts/sec.}
\end{align*}
\]
In the given circuit shown in figure below, the steady state is reached with switch ‘k’ is open. At t=0, switch ‘k’ is closed. For the element values given, determine the value of \( V_{a}(0^-) \) and \( V_{a}(0^+) \).

Solution: At \( t=0 \),

\[ V_{a}(0^-)=V_{b}(0^-)=5 \text{ V} \]
\[ V_{b}(0^-)=5V= V_{b}(0^+) \text{ since voltage across the capacitor cannot change instantaneously} \]

At \( t = 0^+ \), Apply KCL to the given circuit, we get

\[
\frac{(V_{a}-5)}{10} + \frac{(V_{a}-V_{b})}{20} = 0 \quad (1)
\]
\[
\frac{(V_{b}-5)}{10} + \frac{(V_{b}-V_{a})}{20} + C\frac{dV_{b}}{dt} = 0 \quad (2)
\]
From (1), At \( t = 0^+ \),

\[
\frac{[V_{a}(0^+)-5]}{10} + \frac{[V_{a}(0^+)]}{10} + \frac{[V_{a}(0^+)-V_{b}(0^+)]}{20} = 0
\]
\[
\frac{[V_{a}(0^+)-5]}{10} + \frac{[V_{a}(0^+)]}{10} + \frac{[V_{a}(0^+)-5]}{20} = 0
\]
\[
\frac{[2V_{a}(0^+)-10 + 2V_{a}(0^+) + V_{a}(0^+)-5]}{20} = 0
\]
\[
\frac{[5V_{a}(0^+) -15]}{20} = 0
\]
\[ V_{a}(0^+) = \frac{15}{5}=3 \text{ V} \]
[13] In the given circuit, switch ‘k’ is closed at t=0. Prove at t = 0⁺,
\[ \frac{d}{dt}i_1 = \left( \frac{V_0}{R} \right) \left[ \omega \cos \omega t - (\sin \omega t / RC) \right] \]
and
\[ \frac{d}{dt}i_2 = \frac{V_0 \sin \omega t}{L} \]

Solution: At t = 0⁺,
\[ i_1(0⁺) = \frac{V_0 \sin \omega t}{R} \]

and
\[ i_2(0⁺) = 0 \]

Apply KVL to the given circuit at t = 0⁺, we get
\[ V_0 \sin \omega t = R i_1 + \frac{1}{C} \int i_1 \, dt \]  \hspace{1cm} (1)
\[ V_0 \sin \omega t = R i_2 + L \frac{di_2}{dt} \]  \hspace{1cm} (2)

Differentiate equation (1) with respect to t, we get
\[ \omega V_0 \cos \omega t = R \frac{di_1}{dt} + i_1 / C \]

At t = 0⁺,
\[ \omega V_0 \cos (0⁺) = R \frac{di_1(0⁺)}{dt} + i_1(0⁺) / C \]
\[ \frac{di_1(0⁺)}{dt} = \left[ \omega V_0 \cos (0⁺) \right] / R - i_1(0⁺) / RC \]
\[ \frac{di_1(0⁺)}{dt} = \left( \frac{V_0}{R} \right) \left[ \left\{ \omega \cos \omega t (0⁺) \right\} - (\sin \omega t / RC) \right] \]

From equation (2)
\[ V_0 \sin \omega t / R = R i_2 + L \frac{di_2}{dt} \]

At t = 0⁺,
\[ L \frac{di_2(0⁺)}{dt} = V_0 \sin \omega t(0⁺) - R i_2(0⁺) \]

Therefore
\[ \frac{di_2(0⁺)}{dt} = \frac{V_0 \sin \omega t(0⁺)}{L} \]