Extrusion and Injection Molding - Analysis

ver. 1
Overview

• Extrusion and Injection molding
  – Flow in screw
  – Flow in cavity or die

• Injection molding
  – Clamp force
  – Cooling time
  – Ejection force
Extrusion schematic

- Hopper
- Thrust bearing
- Gear reducer box
- Motor
- Screw
- Throat liner
- Barrel heater/cooler
- Thermocouples
- Throat-cooling channel
- Feed zone
- Melting zone
- Melt-pumping zone
- Filter screen
- Breaker plate
- Melt thermocouple
- Adapter
- Die
Injection molding schematic
Flow in screw - Extrusion and Injection molding

- Understood through simple fluid analysis
- Unroll barrel from screw
  - rectangular trough and lid

\[ v = \pi DN \]

\[ v_x \]

\[ v_z \]

\[ H \]

\[ w/\cos\theta \]

w is like normal pitch
w/\cos\theta is like axial pitch
Flow analysis

- Barrel slides across channel at the helix angle
- \( v_z = \) pumping
- \( v_x = \) stirring

\[
v = \pi DN
\]

\( w/\cos\theta \) is like normal pitch
\( w/\cos\theta \) is like axial pitch
Flow rate

• $v_z$ shows viscous traction work against exit pressure

flow rate = $f$(exit pressure, $v_{\text{barrel}}$, $\mu$, d, w, l)
Flow analysis

• Simplify by using Newtonian fluid

• Separate into drag and pressure flows

• Add solutions (superposition)
Drag flow in rectangular channel \( (Q_D) \)

- Simple viscous flow between parallel plates, end effects negligible

\[
v = v_0 \frac{y}{H}
\]

\[
Q_D = v \cdot A = v_0 \cdot \frac{1}{2} \cdot wH
\]
Pressure flow in rectangular channel

- Assumptions
  - no slip at walls
  - melt is incompressible
  - steady, laminar flow
  - end and side wall effects are negligible
Pressure flow in rectangular channel

Equilibrium

\[
\left(p - \left(p + dp\right)\right) \cdot 2y - 2\tau \cdot dz = 0
\]
Pressure flow in rectangular channel

\[(p - (p + dp)) \cdot 2y - 2\tau \cdot dz = 0\]

\[\tau = -y \cdot \frac{dp}{dz}\]

Newtonian fluid

\[\tau = \mu \cdot \dot{\gamma} = \mu \cdot \frac{dv}{dy}\]
Pressure flow in rectangular channel

- Eliminating $\tau$

$$ dv = \frac{-1}{\mu} \cdot \frac{dp}{dz} \cdot y \cdot dy $$

- Integrating and noting

$@ y = +/- H/2, \; v = 0$

$$ v = \frac{1}{\mu} \cdot \frac{dp}{dz} \cdot \left[ \frac{H^2}{8} - \frac{y^2}{2} \right] $$
Total pressure flow ($Q_p$)

\[
Q_p = w \int_{-H/2}^{H/2} v \cdot dy = \frac{wH^3}{12 \mu} \frac{dp}{dz}
\]
Total flow ($Q$)

$$Q = Q_D - Q_p = w \cdot \left[ \frac{v_z H}{2} - \frac{H^3}{12 \mu} \cdot \frac{dp}{dz} \right]$$

- $dp/dz$ set by
  - back pressure on reciprocating screw (injection molding)
  - die resistance (extrusion)
Nomenclature

• $dz = \text{helical length} = \frac{\text{axial length}}{\sin \theta}$
• $v_z = \text{helix velocity} = v_{\text{barrel}} \cdot \cos \theta$
Flow rate

output pressure

flow rate

1. $\omega$
2. $2\omega$
Flow in round die or runner

Same assumptions as above

Equilibrium

\[
\pi \cdot \left[ (r + dr)^2 - r^2 \right] \cdot dp = 2\pi \cdot \left[ (r + dr)(\tau + d\tau) - r\tau \right] \cdot dz
\]
Flow in round die or runner

\[ \pi \cdot [(r + dr)^2 - r^2] \cdot dp = 2\pi \cdot [(r + dr) \cdot (\tau + d\tau) - r\tau] \cdot dz \]

Neglecting HOT

\[ 2\pi r \cdot dr \cdot dp = 2\pi \cdot (\tau \cdot dr + r \cdot d\tau) \cdot dz \]

\[ \frac{dp}{dz} = \frac{\tau \cdot dr + r \cdot d\tau}{r \cdot dr} = \frac{d(\tau r)}{r \cdot dr} \]
Flow in round die or runner

\[
\frac{dp}{dz} = \frac{\Delta p}{L} = C = \frac{d(\tau r)}{r \cdot dr}
\]

\[
d(\tau r) = C r \cdot dr
\]

\[
\int d(\tau r) = \int C r \cdot dr
\]
Flow in round die or runner

\[ \tau r = C \frac{r^2}{2} \]

\[ \tau = \frac{C}{2} r = \frac{\Delta p}{2L} r \]
Flow in round die or runner

- At center, $\tau = 0$
- At edge of tube (R), $\tau = \max$

\[
\tau_{\text{max}} = \frac{\Delta p \cdot R}{2L}
\]

Newtonian fluid

\[
\tau = \mu \frac{du}{dr}
\]
Flow in round die or runner

\[ \dot{\gamma} = \frac{du}{dr} = \frac{\Delta p \cdot r}{2L\mu} \]

finally

\[ u = \frac{\Delta p}{4\mu L} \left( r^2 - R^2 \right) \]
Flow in round die or runner

\[ u = \frac{\Delta p}{4\mu L} \left( r^2 - R^2 \right) \]

\[ Q_p = \int_0^R 2\pi r \cdot u \cdot dr = \frac{\pi \cdot R^4}{8\mu} \cdot \frac{\Delta p}{L} \]
Flow in rectangular die or runner

• as above

\[ Q_p = \frac{wH^3}{12\mu} \cdot \frac{\Delta p}{L} \]
Extrusion

• Pressure generated by screw rotation
  – flow rate through screw = flow rate through die
    \[ Q(\text{extruder}) = Q(\text{die}) \]
  – pressure rise in screw = pressure drop in die
    \[ dp(\text{extruder}) = \Delta p(\text{die}) \]
Extrusion - Ex. 1-1

• Extrude a polymer through a die with dimensions diameter 5 mm, length 40 mm at rate 10 cm/s

• Screw is fixed, barrel rotates

• More data on next page

• Calculate barrel RPM
Extrusion - Ex. 1-2

- polymer density ($\rho$) = 980 kg/m$^3$
- polymer viscosity ($\mu$) = $10^3$ N-s/m$^2$
- barrel diameter ($D$) = 28 mm
- channel width ($w$) = 21 mm
- channel height ($H$) = 4 mm
- helix angle ($\theta$) = 15 degrees
- length of screw ($L$) = 1.25 m
Extrusion - Ex. 1-3

• First, calculate flow rate

\[ Q_{\text{product}} = v \cdot A = 0.1 \times \frac{\pi (0.005)^2}{4} = 1.96 \times 10^{-6} \, m^3 / s \]

\[ Q_{\text{screw}} = w \cdot \left[ \frac{v_z H}{2} - \frac{H^3}{12 \mu} \cdot \frac{dp}{dz} \right] \]

\[ Q_{\text{die}} = \frac{\pi R^4}{8 \mu} \frac{\Delta p}{L} \quad \text{with} \quad dp = \Delta p \]
Extrusion - Ex. 1-4

• Substituting, equating, solving

\[ Q_{product} = Q_{die} \]

\[ \frac{\pi \left( \frac{0.005}{2} \right)^4}{8 \times 10^3} \frac{\Delta p}{0.04} = 1.96 \times 10^{-6} \]

\[ \Delta p = 5.1 \text{ MPa} \]
Extrusion - Ex. 1-5

• Substituting, equating using $\Delta p$, solving

$$Q_{\text{product}} = Q_{\text{screw}}$$

$$1.96 \times 10^{-6} = 0.021 \left[ \frac{v_z \times 0.004}{2} - \frac{(0.004)^3}{12 \times 10^3} \frac{5.1 \times 10^6}{4.83} \right]$$

$$dz = \frac{l}{\sin \theta} = \frac{1.25}{\sin 15} = 4.83m$$

solving

$$v_z = 49.5 \text{ mm/s}$$
Extrusion - Ex. 1-6

• Solving for RPM

\[ v_{\text{barrel}} = \frac{v_z}{\cos \theta} = \frac{49.5}{\cos 15} = 51.2 \, \text{mm/s} \]

\[ N = \frac{60 \times v_{\text{barrel}}}{\pi \times D} = \frac{60 \times 51.2}{\pi \times 28} = 35 \, \text{RPM} \]
Injection molding cycle

1. To make a shot: use screw (extruder) equation for flow rate \((Q)\) to produce a shot volume \((\text{vol} = Q\cdot t)\).
   - back pressure gives \(dp\) term
   - time \((t)\) bounded by cycle time (upper) and degradation of material (lower)
2. To inject the plastic: use pressure flow equations and injection pressure ($\Delta p$) or injection time ($t$) and volume to be filled (shot volume) to determine flow rate ($Q$) and hence time ($t$) or injection pressure ($\Delta p$) required to fill mold

- injection time ($t$) will be limited by freezing of plastic and degradation of material
Injection Molding - Ex. 2-1

• Injection mold a polymer in a steel tool
• Model the sprue, runner and part as a cylinder of diameter 10 mm, length 150 mm
• Determine the screw RPM to make a shot in less than 3 seconds (screw rotates)
• Determine the injection pressure to make the part in 2 seconds
Injection Molding - Ex. 2-2

- polymer density ($\rho$) = 980 kg/m$^3$
- polymer viscosity ($\mu$) = $10^3$ N-s/m$^2$
- barrel diameter (D) = 28 mm
- channel width (w) = 21 mm
- channel height (H) = 4 mm
- helix angle ($\theta$) = 15 degrees
- length of screw (L) = 1.25 m
Injection Molding - Ex. 2-3

- Screw RPM calculation
- Back pressure = 15 MPa
- Assume 3 seconds to make shot
- Calculate Q

\[
Q = \frac{vol}{time} = \frac{\pi r^2 l}{time} = \frac{\pi \cdot (5)^2 \cdot (150)}{3} = 3,927 \, mm^3 / s
\]
Injection Molding - Ex. 2-4

• Screw RPM calculation

\[ Q_{screw} = w \cdot \left[ \frac{v_z H}{2} - \frac{H^3}{12 \mu} \cdot \frac{dp}{dz} \right] \]

\[ v_z = v_{screw} \cos \theta \]

\[ dz = \frac{l}{\sin \theta} \]

\[ v_{screw} = \frac{\pi DN}{60} \]

\[ D = 28 - 2 \times 4 = 20 \text{mm} \]

\[ D = \text{barrel diameter} - 2 \times \text{channel height} \]
Injection Molding - Ex. 2-5

• Substituting values, solving

\[ 3,927 = 21 \cdot \left[ \frac{\pi \times 20 \times \frac{N}{60} \times \cos 15 \times 4}{2} - \frac{4^3}{12 \times 10^3} \cdot \frac{15 \times 10^6}{\sin 15} \right] \]

\[ N = 101 \text{ RPM} \]
Injection Molding - Ex. 2-6

- Injection pressure calculation
- Part injection is pressure driven

\[
Q = \frac{vol}{time} = \frac{\pi (5)^2 (150)}{2} = 5,891 \text{mm}^3 / s
\]

\[
Q_{\text{mold}} = \frac{\pi R^4 \Delta p}{8 \mu L}
\]
Injection Molding - Ex. 2-7

• Substituting, equating, solving

\[ 5,891 = \frac{\pi \times (5)^4}{8 \times 10^3} \frac{\Delta p}{150} \]

\[ \Delta p = 3.6 \text{ MPa} = 522 \text{ psi} \]
Power law viscosity

$$\mu(\dot{\gamma}) = k \cdot \dot{\gamma}^{n-1}$$

$$\tau = \mu \cdot \dot{\gamma} = k \cdot \dot{\gamma}^n$$

$k$, $n$ are consistency and power law index
Non-Newtonian, pressure driven flow in rectangular channel

- NB: drag flow analysis is similar to the following

\[
v = v_0 \cdot \left[1 - \left(\frac{2y}{H}\right)^n\right]
\]

\[
Q = 2wv_0 \cdot \int_0^\frac{H}{2} \left[1 - \left(\frac{2y}{H}\right)^n\right] dy
\]
Non-Newtonian, pressure driven flow in rectangular channel

\[ Q = w \cdot \left( \frac{\Delta p}{k \cdot L} \right)^{\frac{1}{n}} \cdot \frac{2n}{2n + 1} \left( \frac{H}{2} \right)^{\frac{2n+1}{n}} \]

\[ v_{ave} = \frac{Q}{wH} = \left( \frac{\Delta p}{k \cdot L} \right)^{\frac{1}{n}} \cdot \frac{n}{2n + 1} \cdot \left( \frac{H}{2} \right)^{\frac{n+1}{n}} \]
Non-Newtonian, pressure driven flow in round channel

\[ u = \frac{n}{n+1} \cdot \left( \frac{\Delta p}{2k \cdot L} \right)^\frac{1}{n} \cdot R^n \cdot \frac{n+1}{n} \cdot \left[ 1 - \left( \frac{r}{R} \right)^\frac{n+1}{n} \right] \]

\[ Q = \pi \cdot \frac{n}{3n+1} \cdot \left( \frac{\Delta p}{2k \cdot L} \right)^\frac{1}{n} \cdot R^n \cdot \frac{3n+1}{n} \]

\[ v_{ave} = \frac{Q}{\pi \cdot R^2} = \frac{n}{3n+1} \cdot \left( \frac{\Delta p}{2k \cdot L} \right)^\frac{1}{n} \cdot R^n \cdot \frac{n+1}{n} \]
Example – 3-1

• Compare Newtonian and Non-Newtonian, pressure driven fluid flow in a rectangular channel

• Given
  – $H = 2 \text{ mm}$, $w = 15 \text{ mm}$, $L = 50 \text{ mm}$
  – $Q = 60 \text{ cm}^3/\text{s} = 6 \times 10^{-5} \text{ m}^3/\text{s}$
  – $\mu = 100 \text{ Pa-s} @ d\gamma/dt = 3000/\text{s}$ (Newtonian viscosity)
    • $k = 12198$, $n = 0.4$
Example – 3-2

First, determine the Newtonian flow properties

\[ v_{ave} = \frac{Q}{wH} = \frac{6 \times 10^{-5}}{0.015 \cdot 0.002} = 2 \frac{m}{s} \]

\[ \Delta p = \frac{12 \mu LQ}{wH^3} = \frac{12 \cdot 100 \cdot 0.05 \cdot 6 \times 10^{-5}}{0.015 \cdot (0.002)^3} = 30 \text{ MPa} \]
Example – 3-3

\[ v = \frac{1}{\mu} \cdot \frac{\Delta p}{L} \cdot \left[ \frac{H^2}{8} - \frac{y^2}{2} \right] \]

\[ v_{\text{max}} = v \bigg|_{y=0} = \frac{\Delta p \cdot H^2}{8 \mu L} = \frac{30 \times 10^6 \cdot (0.002)^2}{8 \cdot 100 \cdot 0.05} = 3 \frac{m}{s} \]

(max at y=0 because this gives the greatest value)
Example – 3-4

• For non-Newtonian flow, determine the $\Delta p$ needed for $Q = 6 \times 10^{-5} \text{ m}^3/\text{s}$ and $v_{\text{ave}} = 2 \text{ m/s}$.

\[
v_{\text{ave}} = \frac{Q}{wH} = \left( \frac{\Delta p}{k \cdot L} \right)^{\frac{1}{n}} \cdot \frac{n}{2n+1} \cdot \left( \frac{H}{2} \right)^{\frac{n+1}{n}}
\]

\[
2 \frac{m}{s} = \left( \frac{\Delta p}{12198 \cdot 0.05} \right)^{\frac{1}{0.4}} \cdot \frac{0.4}{2 \cdot 0.4 + 1} \cdot \left( \frac{0.002}{2} \right)^{\frac{0.4+1}{0.4}}
\]
Example – 3-5

solving

\[ \Delta p = 23.3 \text{ MPa} \]

and

\[ \frac{\Delta p_{\text{non-Newtonian}}}{\Delta p_{\text{Newtonian}}} = \frac{23.3}{30} \approx 0.78 \]
Example – 3-6

• For non-Newtonian flow, determine $Q$ for $\Delta p = 30$ MPa.

\[
v_{ave} = \left( \frac{30 \times 10^6}{12198 \cdot 0.05} \right)^{\frac{1}{0.4}} \cdot \frac{0.4}{2 \cdot 0.4 + 1} \cdot \left( \frac{0.002}{2} \right)^{\frac{0.4+1}{0.4}} = 3.77 \frac{m}{s}
\]

\[
Q = v_{ave} \cdot w \cdot H = 3.77 \cdot 0.015 \cdot 0.002 = 11.3 \times 10^{-5} \frac{m^3}{s}
\]

\[
\frac{Q_{non-Newtonian}}{Q_{Newtonian}} = \frac{11.3 \times 10^{-5}}{6 \times 10^{-5}} = 1.88
\]
Example – 3-7

- One can see the effect of shear-thinning
  - reduction in pressure needed to maintain a flow
  - increase in flow with a constant pressure
Clamp force

• Typically 50 tons/oz of injected material
• Can be approximated by
  – injection pressure x projected area of part at parting line
Cooling in a mold

• Assume 1-D heat conduction
• Assume mold conducts much better than plastic (Biot > 1)
• Center temperature important

\[ \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \]

T = temperature
t = time
\( \alpha = \frac{k}{\rho c} \)}
Cooling in a mold

\[ Fo = \frac{\alpha t}{x^2} \quad Bi = \frac{hx}{k} \quad \xi = \frac{x}{l} \]

\[ \Theta = \frac{T_E - T_W}{T_M - T_W} \]

\[ \Theta = ejection \ temp \]
\[ T_M = injection \ temp \]
\[ T_W = mold \ wall \ temp \]
\[ 2l = thickness \ of \ part \]

\[ \frac{\partial^2 \Theta}{\partial \xi^2} = \frac{\partial \Theta}{\partial Fo} \]
Cooling in a mold

- Solution
  - must be approximated or solved numerically

\[
\Theta(\xi, Fo) = \frac{4}{\pi} \sum_{n, \text{odd}}^{\infty} \left[ \frac{1}{n} \exp\left( -\left( \frac{n\pi}{2} \right)^2 Fo \right) \right] \sin\left( \frac{n\pi}{2} \xi \right)
\]
Minimum cooling time - $t_c$

Approximation for time taken ($t_c$) for center of flat sheet (thickness, $2l$) to reach ejection temperature ($T_E$)

$$t_c = \frac{4l^2}{\pi^2 \alpha} \cdot \ln \left| \frac{4}{\pi} \cdot \left( \frac{T_M - T_W}{T_E - T_W} \right) \right|$$
Minimum cooling time - Ex. 4-1

- $\alpha =$ thermal diffusivity $\sim 10^{-7}$ m$^2$/s
- $2l =$ plate thickness $\sim 3 \times 10^{-3}$ m
- $T_W =$ mold wall temperature $\sim 50^\circ$C
- $T_M =$ melt temperature $\sim 250^\circ$C
- $T_E =$ ejection temperature $\sim 100^\circ$C
- Minimum cooling time for the center line to reach $T_E$
  - $t_c \sim 15$ sec.
Minimum cooling time - $t_c$

- Approximation for cylinder (radius = $r$), solved similarly to the plate

\[
Fo = \frac{\alpha t}{r^2}
\]

\[
t_c = \frac{1.7r^2}{\pi^2\alpha} \cdot \ln\left|1.7 \cdot \left(\frac{T_M - T_W}{T_E - T_W}\right)\right|
\]
Non-isothermal flow

- Flow rate characteristic time constant:
  \[ \tau = \frac{1}{t} \sim \frac{V}{L_x} \]

- Heat transfer rate characteristic time constant:
  \[ \tau = \frac{1}{t} \sim \frac{\alpha}{L_z^2} \]

\[
\frac{\text{Flowrate}}{\text{Heat transferrate}} \sim \frac{V \cdot L_z^2}{\alpha \cdot L_x} = \frac{V \cdot L_z}{\alpha} \cdot \frac{L_z}{L_x}
\]

- Small numbers give short shots
  - thick runners needed
  - ratio should be greater than one for filling
Non-isothermal flow

\[
\text{Flowrate} \sim \frac{0.01 \text{m/s} \times 0.0015 \text{m}}{10^{-7} \text{m}^2 / \text{s}} \times \frac{0.0015 \text{m}}{0.1 \text{m}} \approx 2.25
\]

So, the mold should fill.
Limits on ejection temperature

- Plastic must be cool enough to withstand ejection force from ejection pins without breaking
- Plastic must be cool enough so that upon further cooling will not warp
Ejection force

- Ejection pins force the part out of the mold after the part has cooled and solidified enough.
- The part will shrink onto any cores, leading to an interference fit.
- Model as a thin walled cylinder with closed ends (plastic part) on a rigid core (metal mold).
Thin-walled cylinder with closed ends

\[ \sigma_t = \frac{pd}{2t} = \sigma_1 \]

\[ \sigma_a = \frac{pd}{4t} = \sigma_2 \]

\[ \sigma_r = 0 = \sigma_3 \]
Biaxial strain

\[ \varepsilon_1 = \frac{\sigma_1}{E} - \frac{\nu \sigma_2}{E} = \frac{pd}{2tE} - \frac{pd}{4tE} \]

\[ \varepsilon_1 = \frac{p}{E} \cdot \left( \frac{d}{2t} - \frac{d}{4t} \nu \right) \]

\[ \varepsilon_1 = \alpha \cdot \Delta T \]
Ejection force

\[ p = \frac{E \cdot \alpha \cdot \Delta T}{\left( \frac{d}{2t} - \frac{d}{4t} v \right)} \]

\[ F_{ejection} = \mu \cdot p \cdot A \]

\[ F_{ejection} = \frac{\mu \cdot E \cdot A \cdot \alpha \cdot \Delta T}{\left( \frac{d}{2t} - \frac{d}{4t} v \right)} \]
Nomenclature

- $A = \text{area}$
- $d = \text{core diameter}$
- $E = \text{Young's modulus}$
- $p = \text{pressure}$
- $t = \text{part thickness}$

- $\alpha = \text{thermal expansion coefficient}$
- $\Delta T = \text{temperature differential}$
- $\nu = \text{Poisson's ratio}$
- $\mu = \text{friction coefficient}$
Summary

• Extrusion and Injection molding
  – Flow in screw
  – Flow in cavity or die

• Injection molding
  – Clamp force
  – Cooling time
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